Mathematical model of knee joint degenerative dystrophic changes formation in its flexion contracture at late stages of rheumatoid arthritis

Abstract. Background. Rheumatoid arthritis (RA) is an immunomodulatory, chronic inflammatory disease accompanied by the proliferation and articular cartilage destruction that cause disability. Biomechanical factors in RA can play an essential role in the start and progress of the degenerative processes within the joint that are secondary to the inflammatory process. Materials and methods. A solid simulation 3D-model of the knee joint was created that contained both tibia and fibula, the femur bone, femoral condyle cartilage and tibial plateau cartilage, menisci. It was done for further analytical calculations and finite element modeling calculations. Analytical calculations are based on the data of previous studies of quantitative evaluation of the knee joint contracture (compliance of contracture to the external corrective action) and on calculations data of the joint forces that manifest in ambulation of the patient with RA. Results. The created simulation computer 3D-model of a knee joint with its flexion contracture at late stages of RA shows that the forced passive flexion in the knee joint to 6°, the load on the condyles of the tibial plateau increased by 12.8 %, and as a result of forced passive flexion in the knee joint to 3°, the load on the condyles of the tibial plateau increased by 95.2 %, which is critical and may cause degradation of the bone tissue in the contact area. Conclusions. Increase of load on the lateral areas of the tibial plateau and, correspondingly, the tensions on the contact areas of the femur bone condyles with the tibial plateau, may contribute to the progress of the clinical picture of RA with the increase of arthrofibrosis and osteoarthritis events, particularly in the lateral areas of the knee joint.

Keywords: rheumatoid arthritis; knee joint; flexion contracture; mathematic modelling; finite element method; load-deformation state.

Introduction

Rheumatoid arthritis (RA) is an immunomodulatory, chronic inflammatory disease accompanied by the proliferation and articular cartilage destruction that cause disability. The etiopathogenesis of RA is still not clear, however, several stages of its pathophysiology were discovered, its key feature being inflammatory synovitis [1]. Although, historically the cartilage is regarded as “an innocent bystander” recent data suggest that cartilage degradation in RA is related to the imbalance of anabolic and catabolic activity of the joint chondrocytes, indirectly connected to the synovitis and arthritis. In addition to inflammation, the metabolic activity of the chondrocytes is also affected by the biophysical factors (mechanical stress) [2]. In particular, the biomechanical factors in RA can play an essential role in the start and progress of the degenerative processes within the joint that are secondary to the inflammatory process. However, the sequence of biomechanical and biochemical processes regulating these events in vivo, is still vague [3]. Knee joint lesion in RA is accompanied by the formation of contractures and the development of discordant deformities in the lower limbs, which in turn leads to a partial or complete loss of a limb function [4]. An objective assessment of the functional status of the joint and adjacent tissues is a prerequisite for identification of the treatment prospect and efficiency management of rehabilitation measures [5]. However, the technological parameters and modes of operation of modern goniometers used by clinicians do not always allow differentiated action upon the inherent biomechanical properties of the joints, direct monitoring of certain morpho-functional manifestations of the contracture (stiffness, instability, partial atrophies of the muscle and adjacent tissues), which reduces the effectiveness of existing devices [6].
Materials and methods

In the early stage, a solid simulation 3D-model of the knee joint was created that contained both tibia and fibula, the femur bone, femoral condyle cartilage and tibial plateau cartilage, menisci. It was done for further analytical calculations and finite element modeling calculations (Fig. 1).

Analytical calculations are based on the data of previous studies of quantitative evaluation of the knee joint contracture (compliance of contracture to the external corrective action) and on calculations data of the joint forces that manifest in ambulation of the patient with RA. Determination of tension and load in the knee joint was performed with flexion-extension contracture of 30°. Measurement of compliance indicators of the knee joint contracture regarding the external corrective action yielded average values of the passive motion angle \( \Delta \alpha \), which is the angle of position change in the joint as a result of graduated force \( F = 50 \) N: in the direction of flexion — \( \Delta \alpha = 3^\circ \), in the direction of extension — \( \Delta \alpha = 6^\circ \).

Joint forces of the 60 kg (600 N) RA patient while walking and with the flexion-extension contracture in 30° position, as well as the normal indicators, obtained from our own simulation musculoskeletal modelling using the AnyBody software, are presented in Table 1.

Results

Several calculation diagrams were developed (Fig. 2) to solve the problem. For illustrative purposes, all dimensions and active forces are presented in full size for a specific model.

Measurements of contact surface on the tibial plateau were performed on a solid 3D-model using SolidWorks software tools (Fig. 3).

According to the curvature radius of the medial and lateral condyles, the area of contact on the medial condyle of the tibial plateau was 434 mm\(^2\), and the area of contact on the lateral condyle of the tibial plateau — 255 mm\(^2\). Both areas of contact in flexion position of the knee joint of 30° were insignificantly shifted backwards.

The following assumptions and limitations were introduced to solve these problems. For analytical calculations they do not account for presence of soft tissue structures in the area of contact. Contact takes place between the bones.

As the posterior surface of the joint capsule has fibrotic changes in it, additional center of rotation in the knee joint arises, and is shifted backwards extraarticularly during flexion to the following distance:

\[
L_5 = 51 \text{ mm } \sin \alpha / 2,
\]

where \( \alpha \) is the angle between the axis of the femur bone and additional hinge center of rotation.
Assuming that the femur bone in the problem is immovable (tightly fixed). The area of contact surfaces does not change in flexion. Distribution of force between the medial and lateral condyles is 50/50.

**Calculation of load on the joint surface of the tibial plateau with forced passive flexion by 6° in the knee joint with the force of 50 N.** According to the calculations, total force was calculated by the formula:

$$\Sigma F_{\text{total}} = \int_{31}^{25} F_{\text{med}} \cdot \sin 30 \cdot \sin x \, dx,$$

where $F_{\text{med}}$ is the total force that acts in the knee joint with the flexion contracture in a 30° position, $x$ is the angle of action of total force on the tibial plateau.

The next step was the calculation of the total force at additional passive movement of the limb segment within the flexion range from 1 to 6°.

The following ratios were used to determine the tension indicators on the medial and lateral condyles of the tibial plateau:

$$\sigma_m = \frac{F_m}{S_m},$$

where $F_m$ is the force that acts on the medial condyle respectively ($i = 6$), $S_m = 434 \text{ mm}^2$ is the area of contact of the medial condyle; and

$$\sigma_l = \frac{F_l}{S_l},$$

where $F_l$ is the force that acts on the lateral condyle respectively ($i = 6$); $S_l = 255 \text{ mm}^2$ is the area of contact of the lateral condyle.

Resulting indicators of forces and tensions are presented in Table 2.

Thus, as a result of passive flexion in the knee joint to 6°, the load on the tibial plateau condyles has increased by 12.8 % (Fig. 4).

The concept of “admissible tensions” $[\sigma]$ was introduced, equal to:

$$[\sigma] = \min \left\{ \frac{R^T}{n_m}, \frac{R^T}{n_m} \right\},$$

where $n_m = 2.6$, for the bone and $n_m = 1.5$ [11].

Compressive strength limits in this case vary from 17.5 to 93.4 MPa; conservatively (in the most dangerous variant) we have taken 17.5 MPa, accordingly:

$$[\sigma] \min = \frac{R^T}{n_m} = 6.73 \text{ MPa}.$$
Then the condition of strength for the model is satisfied when
\[ \frac{\sigma}{\sigma_{\text{max}}} \geq n = 1. \]

Based on this, we can obtain minimal strength margin
\[ n_{\text{min}} = \frac{6.93}{5.02} = 1.38 \]
for the position of passive flexion of 5° on the lateral condyle, that approaches the critical one (Fig. 5).

**Calculation of load on the joint surface of the tibial plateau with forced passive extension at 3° in the knee joint with the force of 50 N.** To solve this problem, let us introduce the following assumptions and limitations similarly to the previous problem. Total force that acts in the knee joint at extension contracture in the position of 30° is the same:
\[ F_{\text{total}} = 2072.4 \text{ N}. \]

The next step was the calculation of the total force at additional passive movement of the limb segment within the extension range from 1 to 3°. The obtained indicators are presented in Table 3.

Thus, as a result of passive extension in the knee joint to 3°, load on the condyles of the tibial plateau increased by 95.2% (Fig. 6).

Based on this, we can obtain minimal strength margin
\[ n_{\text{min}} = \frac{6.93}{7.93} = 0.87 \]
for the position of passive flexion to 3° in the lateral condyle, which is critical for this position and may cause degradation of the bone tissue in the contact area.

**Calculation of load on the joint surface of the tibial plateau when standing with one support under the body weight 60 kg (600 N) within the flexion-extension contracture in a position of 30°.** Total force acting on the knee joint is as follows:
\[ \sum F_{\text{total}} = \sqrt{F_x^2 + F_y^2 + F_z^2} = 2756 \text{ N}. \]

Direction and values of the intraarticular forces within the flexion-extension contracture of the knee joint in a position of 30° are presented in Fig. 9.

**Table 3. Results of passive extension in the knee joint with the force of 50 N**

<table>
<thead>
<tr>
<th>Extension angle, degrees</th>
<th>( F_{\text{total}} ) (N)</th>
<th>( \sigma_m ) (MPa)</th>
<th>( \sigma_l ) (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>2072.36</td>
<td>2.39</td>
<td>4.06</td>
</tr>
<tr>
<td>1</td>
<td>2389.69</td>
<td>2.75</td>
<td>4.69</td>
</tr>
<tr>
<td>2</td>
<td>3277.36</td>
<td>3.78</td>
<td>6.43</td>
</tr>
<tr>
<td>3</td>
<td>4046.58</td>
<td>4.66</td>
<td>7.93</td>
</tr>
</tbody>
</table>
Let us look at the tension in normal conditions at the same positions of the knee joint, for comparison.

\[
\sum F_{\text{total}} = \sqrt{F_x^2 + F_y^2 + F_z^2} = 2016.7 \text{ N}.
\]

Considering the contact areas on the tibial plateau condyles, the tension on them was determined:

\[
\sigma_m = \frac{F_m}{S_m} = 2.32 \text{ MPa},
\]
\[
\sigma_l = \frac{F_l}{S_l} = 3.95 \text{ MPa},
\]

where \(S_m = 434 \text{ mm}^2\) is the contact area of the medial condyle, \(S_l = 255 \text{ mm}^2\) is the contact area of the lateral condyle.

Comparative analysis of the tension indicators in ambulation within the flexion-extension contracture of the knee joint in a position of 30° and in normal conditions is presented in Fig. 11.

**Discussion**

With the development of clinical analysis methods of ambulation (3D kinetics and kinematics), a necessary tool emerged to search for differences between the pathological patterns of ambulation and its normal indices. Computer technologies and software development contribute to collection, analysis, and interpretation of ambulation data, as a tool to study the function of joints in RA [7, 9].

Significant decrease of excursion in thigh joint, knee joint, ankle joint (external-internal rotation, abduction-adduction, flexion-extension) with a contracture in patients with RA causes a complete dysfunction of a support [10]. Under such conditions, including marked pain reaction for the patient, the loads on all elements of the large joints and muscle groups of the lower limbs increase significantly [11].

Increase of the mechanical load, in the setting of an inflammatory process, capsular and ligamentous disorders, cartilage degradation, subchondral bone changes and muscle imbalance contribute to the progress of joint and muscle contractures, as well as arthritic events in the large joints of the lower limbs, including in the setting of the arising erosions of articular surfaces [12]. Therefore, the
importance of mechanical factors in the destructive cascade of the processes in RA is beyond doubt [10].

Decreased muscle strength, discovered in the clinical study, is associated with the inflammatory process activity, radiologic abnormalities, and functional disorder degrees [13]. Contracture (Latin contracture — tightening, narrowing) is a limitation of the normal functioning of the joint, related to pathological changes in the surrounding tissues: skin cover, muscles, tendons, ligaments, articular capsule, articular surfaces of bones (M24.5 according to ICD-10).

The major sign of the contracture is a limitation of passive and active movements in the joint. In addition to the limitation of joint movements, any contracture is characterized by an early progression of muscle atrophy, which is evident by a decrease in their volume, strength, and endurance [14, 15]. The term “contracture position” refers to the forced position of the joint due to the limitation of movements in it. Depending on the degree of the joint mobility limitation, the contracture can be with preservation of the motion range and possibility of its examination (goniometry); rigid — with a lack of flexibility and compliance, the range of motion in the joint is about 5°, which cannot be determined by goniometer; ankylosing — a complete loss of motor activity in the joint [16, 17].

Understanding the intensity of the biomechanical load on the articular surfaces in the setting of contracture of the joints of the lower limbs in RA and the participation of the muscle forces in this process, can contribute to the development of new views and approaches to the tactic of therapeutic measures that are specific to each stage of the disease [18].

As can be seen from the obtained results, the tension in the contact area on the lateral condyle of the tibial plateau is increased by 36.71 %, whereas on the medial condyle of the tibial plateau — by 36.64 %, under the conditions of flexion-extension contracture of the knee joint in a position of 30°. To validate the obtained results of mathematical analytical calculations, simulation modelling with calculations of stress-strain condition will be performed in the ANSYS software environment at the next stage. In addition, the effect of cyclic loads on joint surfaces of knee joints in patients with rheumatoid arthritis will be studied. According to the curvature radius of the medial and lateral condyles, the area of contact on the medial condyle of the tibial plateau was 434 mm², and the area of contact on the lateral condyle of the tibial plateau — 255 mm². Both areas of contact in flexion position of the knee joint of 30° were insignificantly shifted backwards.

As a result of forced passive flexion in the knee joint to 6°, the load on the condyles of the tibial plateau increased by 12.8 %, and as a result of forced passive flexion in the knee joint to 3°, the load on the condyles of the tibial plateau increased by 95.2 %, which is critical and may cause degradation of the bone tissue in the contact area.

When standing with one support under the body weight 60 kg (600 N) in conditions of flexion-extension contracture in a position of 30°, an increase of contact tensions on the lateral condyle of the tibial plateau by 36.71 % (5.40 MPa) can be observed; on medial condyle — by 36.64 % (3.17 MPa),
in relation to the values of an intact knee joint (3.95 and 2.32 MPa, respectively).

Conclusions
The importance of mechanical factors in the destructive cascade of the processes in RA is beyond doubt. Decreased muscle strength, discovered in the clinical study, is associated with the inflammatory process activity, radiologic abnormalities, and functional disorder degrees. Understanding the intensity of the biomechanical load on the articular surfaces in the setting of contracture of the joints of the lower limbs in RA, and the participation of the muscle forces in this process, can contribute to the development of new views and approaches to the tactic of therapeutic measures that are specific to each stage of the disease. Increase of load on the lateral areas of the tibial plateau and, correspondingly, the tensions on the contact areas of the femur bone condyles with the tibial plateau may contribute to the progress of the clinical picture of rheumatoid arthritis with the increase of arthrofibrosis and osteoarthrosis events, particularly in the lateral areas of the knee joint.

References

Received 12.04.2023
Revised 30.05.2023
Accepted 02.06.2023
Резюме. Актуальність. Біомеханічні чинники при ревма-тодіному артріті (РА) можуть відіграюти важливу роль в іні-ціюванні та прогресуванні дегенеративних процесів у суглобі, вторинних щодо запального процесу. Проте послідовність біомеханічних і біохімічних процесів, що регулюють ці події в живому організмі, поки недостатньо ясна. Розуміння величин біомеханічних навантажень на суглобові поверхні в умовах згинальної контрактури суглобів нижніх кінцівок при РА може допомогти вирішити завдання створення імітаційної комп'ютерної 3D-моделі колінного суглоба (КоС) при його згинальній контрактурі. Матеріали та методи. За основу аналітичних розрахунків покладено дані попередніх досліджень щодо кількісної оцінки контрактури КоС (підгатуваність контрактури зовнішній кінцевий ризик згинальної фази і та розрахунки суглобових сил, що виникають при ходьбі пацієнта з РА. Визначення напружень та навантажень у суглобах проводили при згинально-розгинальній контрактурі 30°. Результати. Для вирішення цієї задачі здійснено декілька розрахункових схем, на яких зображено всі розміри, діючі сили в натуральну величину для конкретної моделі. Унаслідок промислової пасивної згинання в КоС на 6° навантаження на виростки плато великомілкової кістки збільшилося на 12,8 %, унаслідок пасивного розгинання в КоС на 3° — на 95,2 %, що є критичним та може призводити до легерації кісткової тканини в ділянці контактного зміщення плато великомілкової кістки та, відповідно, напружень у ділянках контактного зміщення плато великомілкової кістки з плато великомілкової кістки може призводити до прогресування клінічної картини РА з посиленням артрофіброзу та остеоартриту в задніх відділах КоС. Висновки. Зростання навантаження на виростки плато великомілкової кістки може призводити до прогресування клінічної картини РА з посиленням артрофіброзу та остеоартриту в задніх відділах КоС.

Ключові слова: ревматоїдний артрит; колінний суглоб; згинальна контрактура; математичне моделювання; метод скінченних елементів; напружено-деформуючий ста